

Hawking Temperature of Dilaton Black Holes from Tunneling

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Abstract

Recently, it has been suggested that Hawking radiation can be derived from quantum tunnelling methods. In this letter, we calculated Hawking temperature of dilatonic black holes from tunnelling formalism. The two semi-classical methods adopted here are: the null-geodesic method proposed by Parikh and Wilczek and the Hamilton-Jacobi method proposed by Angheben *et al.* We apply the two methods to analysis the Hawking temperature of the static spherical symmetric dilatonic black hole, the rotating Kaluza-Klein black hole, and the rotating Kerr-Sen black hole.

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I. INTRODUCTION

S. W. Hawking[1] have been able to derive that black hole can radiate from the event horizon like a black body at the temperature $T = \frac{\kappa}{2\pi}$ using the method of quantum field theory in curved spacetime. Recent proposal of deriving the Hawking temperature via the semi-classical methods have been attracted a lot of interests [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26]. The complex path method[2, 3] was first proposed by K. Srinivasan and T. Padmanabhan, and subsequently developed by many authors [4, 5, 6, 7, 8, 9]. The developed method is called Hamilton-Jacobi method in which the Hawking temperature was obtained from calculating the imaginary part of classical action toward Hamilton-Jacobi equation. Another method called null-geodesic method was proposed by M. K. Parikh and F. Wilczek in [10]. There have been considerable efforts to generalize this method to those of various black hole solutions. In this letter, we will apply the two method mentioned above to analysis the Hawking temperature of dilatonic black hole. Our prime motivation is to understand the applicability of the two methods to dilaton black holes in string theory. The Hawking radiation of three kinds of black holes investigated in this letter have been studied in [27] using the gravitational anomaly cancellation mechanism firstly proposed in [28]. The discussion of thermodynamics of some black holes in dilaton gravity can be found in [29, 30]. Let us begin with the spherical symmetric dilaton black hole.

II. HAWKING TEMPERATURE OF SPHERICAL SYMMETRIC DILATON BLACK HOLE

The action for the dilaton gravity describing the dilaton field coupled to the $U(1)$ gauge field in $(3+1)$ dimensional is subject to the form

$$S = \frac{1}{16\pi} \int dx^4 \sqrt{-g} [R - 2\nabla^\mu\phi\nabla_\mu\phi - e^{-2\alpha\phi} F_{\mu\nu}F^{\mu\nu}] , \quad (1)$$

where ϕ is dilaton field and $F_{\mu\nu} = 2\nabla_{[\mu}A_{\nu]}$ is the $U(1)$ gauge field respectively, with an arbitrary coupling constant α . From the action, the static spherically symmetric solution of

motion equation for the underlying theory can be written as[31]

$$ds^2 = -\frac{\Delta}{R^2}dt^2 + \frac{R^2}{\Delta}dr^2 + R^2(d\theta^2 + \sin^2\theta d\varphi^2) , \quad (2)$$

$$\phi = \frac{\alpha}{1+\alpha^2} \ln\left(1 - \frac{r_-}{r}\right) , \quad (3)$$

$$F = \frac{Q}{r^2}dt \wedge dr , \quad (4)$$

where

$$\Delta = (r - r_+)(r - r_-) , \quad R = r\left(1 - \frac{r_-}{r}\right)^{\alpha/(1+\alpha^2)} .$$

in which the outer and inner horizons are respectively given by

$$r_{\pm} = \frac{1+\alpha^2}{1 \pm \alpha^2} [M \pm \sqrt{M^2 - (1-\alpha^2)Q^2}] . \quad (5)$$

After performing the conformal transformation $\tilde{g}_{\mu\nu} = e^{-2\alpha\phi}g_{\mu\nu}$, the line element becomes the general spherical symmetric form

$$ds^2 = -f(r)dt^2 + \frac{1}{g(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) , \quad (6)$$

where

$$f(r) = \frac{\Delta}{R^2}e^{-2\alpha\phi} , \quad g(r) = \frac{\Delta}{R^2}e^{2\alpha\phi} . \quad (7)$$

Now, we focus on investigating Hawking temperature of the dilatonic black hole from quantum tunnelling process. It should be noted that the conformal transformation does not affect the final result as shown in the following analysis. To apply the null-geodesics method, it is necessary to choose coordinates which are not singular at the horizon. This new coordinate has been systematically studied by Maulik K. Parikh in [32]. Introduce the coordinate transformation

$$dt = dT - \Lambda(r)dr , \quad (8)$$

where the function Λ is required to depend only on r not t . Then the line element becomes

$$\begin{aligned} ds^2 = & -f(r)dT^2 + \left(\frac{1}{g(r)} - f(r)\Lambda(r)\right)dr^2 \\ & + 2f(r)\Lambda(r)dTdr + r^2d\Omega^2 . \end{aligned} \quad (9)$$

Restrict the condition

$$\frac{1}{g(r)} - f(r)\Lambda(r) = 1 . \quad (10)$$

One can obtain the line element in new coordinate

$$ds^2 = -f(r)dT^2 + 2\sqrt{\frac{f(r)(1-g(r))}{g(r)}}dTdr + dr^2 + r^2d\Omega^2. \quad (11)$$

This coordinate system has a number of interesting features. At any fixed time the spatial geometry is flat. At any fixed radius the boundary geometry is the same as that of the original metric.

The radial null geodesics for this metric is given by

$$\dot{r} = \sqrt{\frac{f(r)}{g(r)}}(\pm 1 - \sqrt{1-g(r)}) . \quad (12)$$

The imaginary part of the classical action for an outgoing positive energy particle is

$$\text{Im}S = \text{Im} \int_{r_{in}}^{r_{out}} p_r dr = \text{Im} \int_{r_{in}}^{r_{out}} \int_0^{p_r} dp'_r dr . \quad (13)$$

where r_{in} and r_{out} are the initial and final radii of the black hole, respectively. Assume that the emitted energy $\omega' \ll M$. According to energy conservation, the energy of background spacetime M becomes $M - \omega'$. From Hamilton equation $\dot{r} = \frac{dH}{dp_r}|_r$, the integral can be rewritten as

$$\text{Im}S = \text{Im} \int_{r_{in}}^{r_{out}} \int_M^{M-\omega} \frac{dr}{\dot{r}} dH , \quad (14)$$

where $dH = -d\omega'$ because $H = M - \omega'$. In order to find the Hawking temperature, we can perform a series expansion in ω . The first order gives

$$\begin{aligned} \text{Im}S &= \text{Im} \int_{r_{in}}^{r_{out}} \int_M^{M-\omega} \frac{dr}{\dot{r}(r, M - \omega')} (-d\omega') \\ &= -\omega \text{Im} \int_{r_{in}}^{r_{out}} \frac{dr}{\dot{r}(r, M)} + O(\omega^2) \\ &\simeq \omega \text{Im} \int_{r_{out}}^{r_{in}} \frac{dr}{\dot{r}(r, M)} . \end{aligned} \quad (15)$$

To proceed further we will need to estimate the last integral which can be done by deforming the contour. There is a simple pole at the horizon with a residue $\frac{2}{\sqrt{f'(r_+)g'(r_+)}}$. Hence the imaginary part of the action will be

$$\text{Im}S = \frac{2\pi\omega}{\sqrt{f'(r_+)g'(r_+)}} . \quad (16)$$

Using the WKB approximation the tunneling probability for the classically forbidden trajectory is given by

$$\Gamma = \exp(-2\text{Im}S) . \quad (17)$$

Hartle and Hawking in [33] obtained particle production in the standard black-hole space-times using a semiclassical analysis. The tunneling probability can also be written as

$$\Gamma = \exp(-\beta\omega) , \quad (18)$$

where ω is the energy of the particles and β^{-1} is the Hawking temperature. The higher order terms are a self-interaction effect. For calculating the Hawking temperature, expansion to linear order is all that is required. Comparing the above equation, the hawking temperature is given by

$$T_H = \beta^{-1} = \frac{\sqrt{f'(r_+)g'(r_+)}}{4\pi} . \quad (19)$$

It should be noted that the Hawking temperature is depend on the product of $f'(r)$ and $g'(r)$ which is independent of the conformal transformation. Due to the same reason, it is shown that the Hawking temperature obtained from the Hamilton-Jacobi method is also independent of the conformal transformation. The tunneling method we applied to the spherical symmetric dilaton black hole is not affected by the conformal transformation. For the static spherically symmetric dilatonic black hole in our case, this gives the hawking temperature

$$T_H = \frac{1}{4\pi r_+} \left(1 - \frac{r_-}{r_+}\right)^{(1-\alpha^2)/(1+\alpha^2)} . \quad (20)$$

We now turn to the Hamilton-Jacobi method. Consider a massive scalar field in the static spherically symmetric dilatonic black hole spacetime satisfying Klein-Gordon equation

$$g^{\mu\nu} \nabla_\mu \nabla_\nu \Phi - m^2 \Phi = 0 . \quad (21)$$

By performing the WKB approximation, *i.e.*, expanding the field function as

$$\Phi = \exp\left(-\frac{i}{\hbar}S + \dots\right) , \quad (22)$$

one can obtain Hamilton-Jacobi equation

$$g^{\mu\nu} \partial_\mu S \partial_\nu S + m^2 = 0 . \quad (23)$$

where S is the classical action. For the metric of the form

$$ds^2 = -f(r)dt^2 + \frac{1}{g(r)}dr^2 + h_{ij}dx^i dx^j , \quad (24)$$

where $h_{ij}dx^i dx^j = r^2 d\Omega^2$ in the case of the static spherically symmetric dilatonic black hole spacetime. The Hamilton-Jacobi equation becomes

$$-\frac{(\partial_t S)^2}{f(r)} + g(r)(\partial_r S)^2 + h^{ij}\partial_i S \partial_j S + m^2 = 0 . \quad (25)$$

One can use separation of variables to write the solution of the form

$$S = -Et + W(r) + J(x^i) . \quad (26)$$

As a consequence, one get

$$\partial_t S = -E , \quad \partial_r S = W'(r) , \quad \partial_i S = J_i , \quad (27)$$

where the J_i s are constants. $W(r)$ can be solved

$$W(r) = \int \frac{dr}{\sqrt{f(r)g(r)}} \sqrt{E^2 - f(r)(m^2 + h^{ij}J_i J_j)} . \quad (28)$$

Following [6], we can select the proper spatial distant

$$d\sigma^2 = \frac{dr^2}{g(r)} , \quad (29)$$

where we are only concerned with the radial rays as the null-geodesic method. Performing the near horizon approximation

$$\begin{aligned} f(r) &= f'(r_+)(r - r_+) + \dots , \\ g(r) &= g'(r_+)(r - r_+) + \dots . \end{aligned} \quad (30)$$

we find

$$\sigma = \int \frac{dr}{\sqrt{g(r)}} \simeq \frac{2\sqrt{r - r_+}}{\sqrt{g'(r_+)}} . \quad (31)$$

The imaginary part of the classical action is

$$\begin{aligned} \text{Im}W(\sigma) &= \text{Im} \frac{2}{\sqrt{g'(r_+)f'(r_+)}} \int \frac{d\sigma}{\sigma} \\ &\quad \times \sqrt{E^2 - \frac{\sigma^2}{4}g'(r_+)f'(r_+)(m^2 + h^{ij}J_i J_j)} \\ &= \frac{2\pi E}{\sqrt{g'(r_+)f'(r_+)}} . \end{aligned} \quad (32)$$

So the imaginary part of classical action calculated by the Hamilton-Jacobi method is the same as the previous result from the null geodesics method. Now, we will turn to the rotating dilatonic black hole.

III. HAWKING TEMPERATURE OF KALUZA-KLEIN DILATON BLACK HOLE

The Kaluza-Klein black hole is an exact solution of the dilatonic action with the coupling constant $\alpha = \sqrt{3}$. It is derived by a dimensional reduction of the boosted five-dimensional Kerr solution to four dimensions. The metric is given by[34, 35]

$$\begin{aligned} ds^2 &= -f(r, \theta)dt^2 + \frac{1}{g(r, \theta)}dr^2 - 2H(r, \theta)dtd\varphi \\ &\quad + K(r, \theta)d\varphi^2 + \Sigma(r, \theta)d\theta^2, \\ f(r, \theta) &= \frac{\Delta - a^2\sin^2\theta}{B\Sigma}, \\ g(r, \theta) &= \frac{\Delta}{B\Sigma}, \\ H(r, \theta) &= a\sin^2\theta \frac{Z}{B\sqrt{1-\nu^2}}, \\ K(r, \theta) &= B(r^2 + a^2) + a^2\sin^2\theta \frac{Z}{B}, \\ \Sigma(r, \theta) &= r^2 + a^2\cos^2\theta, \end{aligned} \tag{33}$$

where

$$\begin{aligned} \Delta &= r^2 - 2\mu r + a^2, \\ Z &= \frac{2\mu r}{\Sigma}, \\ B &= \sqrt{1 + \frac{\nu^2 Z}{1 - \nu^2}}. \end{aligned} \tag{34}$$

The dilaton field and gauge potential are respectively

$$\begin{aligned} \phi &= -\frac{\sqrt{3}}{2}\ln B, \\ A_t &= \frac{\nu Z}{2(1 - \nu^2)B^2}, \\ A_\varphi &= -\frac{a\nu Z\sin^2\theta}{1\sqrt{1 - \nu^2}B^2}. \end{aligned} \tag{35}$$

The physical mass M , the charge Q , and the angular momentum J are expressed by the boost parameter ν , mass parameter μ , and specific angular momentum a , as

$$\begin{aligned} M &= \mu \left[1 + \frac{\nu^2}{2(1 - \nu^2)} \right], \\ Q &= \frac{\mu\nu}{1 - \nu^2}, \\ J &= \frac{\mu a}{\sqrt{1 - \nu^2}}. \end{aligned} \tag{36}$$

The outer and inner horizons are respectively given by

$$r_{\pm} = \mu \pm \sqrt{\mu^2 - a^2} . \quad (37)$$

The metric can be written as

$$\begin{aligned} ds^2 &= -F(r, \theta)dt^2 + \frac{1}{g(r, \theta)}dr^2 \\ &\quad + K(r, \theta)\left(d\varphi - \frac{H(r, \theta)}{K(r, \theta)}dt\right)^2 + \Sigma(r, \theta)d\theta^2 , \end{aligned} \quad (38)$$

where

$$\begin{aligned} F(r, \theta) &= f(r, \theta) + \frac{H^2(r, \theta)}{K(r, \theta)} \\ &= \frac{\Delta[(1 - \nu^2)\Sigma + 2\mu\nu^2r]}{B[(1 - \nu^2)\Sigma\Delta + 2\mu r(r^2 + a^2)]} . \end{aligned} \quad (39)$$

At the horizon, one have

$$\frac{H(r_+, \theta)}{K(r_+, \theta)} = \frac{a\sqrt{1 - \nu^2}}{r_+^2 + a^2} = \Omega_H . \quad (40)$$

Because the metric depends on the angle θ , we will apply the method developed in [22, 23] to continue. We will first fix the angle θ , and then show that the final result is independent of the angle θ . The metric near the horizon for fixed $\theta = \theta_0$ is

$$\begin{aligned} ds^2 &= -F_r(r_+, \theta_0)(r - r_+)dt^2 + \frac{dr^2}{g_r(r_+, \theta_0)(r - r_+)} \\ &\quad + K(r_+, \theta_0)d\chi^2 , \end{aligned} \quad (41)$$

where $F_r(r, \theta)$ denotes the partial differential of $F(r, \theta)$, $g_r(r, \theta)$ denotes the partial differential of $g(r, \theta)$, and $d\chi = d\varphi - \Omega_H dt$ is new coordinate parameter. This metric is well behaved for all θ_0 and is of the same form as the spherical symmetric metric (6) in the last section. To see this point more clearly, one can use the drag coordinate like this

$$\frac{d\varphi}{dt} = \Omega_H , \quad (42)$$

which just means $d\chi = 0$. Then, the metric can further reduce to the form

$$ds^2 = -F_r(r_+, \theta_0)(r - r_+)dt^2 + \frac{dr^2}{g_r(r_+, \theta_0)(r - r_+)} . \quad (43)$$

According to the procedure in the last section, one can easily obtain the final result bying considering the massive particle escaping from the horizon

$$T_H = \frac{\sqrt{F_r(r_+, \theta_0)g_r(r_+, \theta_0)}}{4\pi} . \quad (44)$$

Direct calculation for $F_r(r_+, \theta_0)$ and $g_r(r_+, \theta_0)$ gives

$$\begin{aligned} F_r(r_+, \theta_0) &= \frac{\sqrt{1 - \nu^2} \Delta_r(r_+)}{4\mu^2 r_+^2} \sqrt{\Sigma(r_+, \theta_0)} \\ &\quad \times \sqrt{(1 - \nu^2)\Sigma(r_+, \theta_0) + 2\mu\nu^2 r_+}, \\ g_r(r_+, \theta_0) &= \frac{\Delta_r(r_+)}{\sqrt{\Sigma(r_+, \theta_0)}} \\ &\quad \times \frac{1}{\sqrt{(1 - \nu^2)\Sigma(r_+, \theta_0) + 2\mu\nu^2 r_+}}. \end{aligned} \quad (45)$$

Although $F_r(r_+, \theta_0)$ and $g_r(r_+, \theta_0)$ each depend on θ_0 , their product gives the Hawking temperature

$$T_H = \frac{1}{2\pi} \frac{\sqrt{1 - \nu^2} \sqrt{\mu^2 - a^2}}{(r_+^2 + a^2)}. \quad (46)$$

which is independent of θ_0 .

Now, we turn to the Hamilton-Jacob method to calculate the Hawking temperature. According to metric (41), the action can be assumed to of the form

$$I = -Et + J\varphi + W(r, \theta_0). \quad (47)$$

In terms of the relation $\chi(r_+) = \varphi - \Omega_H t$, the classical action can be written as

$$I = -(E - \Omega_H J)t + J\chi + W(r, \theta_0). \quad (48)$$

It is easy to obtain the imaginary part of action. In fact, the similarity between the metric in this section and the metric in the last section reminds us that one can just replace E with $(E - \Omega_H J)$ to obtain the imaginary part of classical action. The result is given by

$$\text{Im}W(r, \theta_0) = \frac{2\pi(E - \Omega_H J)}{\sqrt{F_r(r_+, \theta_0)g_r(r_+, \theta_0)}}. \quad (49)$$

This in turn gives the same temperature

$$T_H = \frac{1}{2\pi} \frac{\sqrt{1 - \nu^2} \sqrt{\mu^2 - a^2}}{(r_+^2 + a^2)}. \quad (50)$$

The two methods used in this letter are also valid to the rotating Kaluza-Klein black hole. Now, we will turn to a more general case to analysis this validity.

IV. HAWKING TEMPERATURE OF KERR-SEN DILATON BLACK HOLE

The Kerr-Sen black hole[36] is a solution to the low-energy effective action in heterotic string theory. The action is

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} [R - 2\nabla^\mu\phi\nabla_\mu\phi - e^{-2\phi}F_{\mu\nu}F^{\mu\nu} - \frac{1}{12}e^{-4\phi}H^2]. \quad (51)$$

where H is the three-form axion field and the coupling constant $\alpha = 1$. Sen adopted the solution generating technique to obtain a new solution from the uncharged Kerr solution. The metric is given by

$$\begin{aligned} ds^2 &= -f(r,\theta)dt^2 + \frac{1}{g(r,\theta)}dr^2 - 2H(r,\theta)dtd\varphi \\ &\quad + K(r,\theta)d\varphi^2 + \Sigma(r,\theta)d\theta^2, \\ f(r,\theta) &= \frac{\Delta - a^2\sin^2\theta}{\Sigma}, \\ g(r,\theta) &= \frac{\Delta}{\Sigma}, \\ H(r,\theta) &= \frac{2\mu r \cosh^2\beta \sin^2\theta}{\Sigma}, \\ K(r,\theta) &= \frac{\Lambda \sin^2\theta}{\Sigma}, \\ \Sigma(r,\theta) &= r^2 + a^2\cos^2\theta + 2\mu r \sinh^2\beta, \end{aligned} \quad (52)$$

where

$$\begin{aligned} \Delta &= r^2 - 2\mu r + a^2, \\ \Lambda &= (r^2 + a^2)(r^2 + a^2\cos^2\theta) + 2\mu r a^2 \sin^2\theta \\ &\quad + 4\mu r(r^2 + a^2)\sinh^2\beta + 4\mu^2 r^2 \sinh^4\beta. \end{aligned} \quad (53)$$

The dilaton field, axion field, and gauge potential are respectively given by

$$\begin{aligned} \phi &= \frac{1}{2}\ln\frac{\Sigma}{r^2 + a^2\cos^2\theta}, \\ B_{t\varphi} &= 2a\sin^2\theta \frac{\mu r \sinh^2\beta}{\Sigma}, \\ A_t &= \frac{\mu r \sinh 2\beta}{\sqrt{2}\Sigma}, \\ A_\varphi &= \frac{a\mu r \sinh 2\beta \sin^2\theta}{\sqrt{2}\Sigma}. \end{aligned} \quad (54)$$

The mass M , the charge Q , and the angular momentum J are given as

$$\begin{aligned} M &= \frac{\mu}{2}(1 + \cosh 2\beta) , \\ Q &= \frac{\mu}{\sqrt{2}} \sinh^2 2\beta , \\ J &= Ma . \end{aligned} \quad (55)$$

The outer and inner horizons is determined as

$$r_{\pm} = \mu \pm \sqrt{\mu^2 - a^2} . \quad (56)$$

As discussed in last section, the metric can be written in the form (37). Now, the function $F(r, \theta)$ is given by

$$F(r, \theta) = \frac{\Delta\Sigma}{\Lambda} . \quad (57)$$

The angular velocity is

$$\Omega_H = \frac{H(r_+, \theta)}{K(r, \theta)} = \frac{a}{(r_+^2 + a^2)} \frac{1}{\cosh^2 \beta} . \quad (58)$$

As shown in the last section, the same procedure of applying the two methods in this black hole solution will give the same final result

$$T_H = \frac{\sqrt{F_r(r_+, \theta_0)g_r(r_+, \theta_0)}}{4\pi} . \quad (59)$$

Direct calculation for $F_r(r_+, \theta_0)$ and $g_r(r_+, \theta_0)$ gives

$$\begin{aligned} F_r(r_+, \theta_0) &= \frac{\Delta_r(r_+)(2\mu r_+ \cosh^2 \beta - a^2 \sin^2 \theta_0)}{4\mu^2 r_+^2 \cosh^4 \beta} , \\ g_r(r_+, \theta_0) &= \frac{\Delta_r(r_+)}{2\mu r_+ \cosh^2 \beta - a^2 \sin^2 \theta_0} . \end{aligned} \quad (60)$$

Although $F_r(r_+, \theta_0)$ and $g_r(r_+, \theta_0)$ each depend on θ_0 , their product gives the Hawking temperature

$$T_H = \frac{1}{2\pi} \frac{\sqrt{\mu^2 - a^2}}{(r_+^2 + a^2) \cosh^2 \beta} . \quad (61)$$

which is independent of θ_0 . In this section, we see that the null geodesics method and the Hamilton-Jacobi method are also valid to the rotating Kerr-Sen dilaton black hole solution.

V. CONCLUSION

In this letter, we have succeeded in extending the semi-classical methods to calculate the Hawking temperature of dilatonic black holes in string theory. The results is consistent with the underlying unitary theory.

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